Machine learning discovers parsimonious equations governing incoherent emission steering from semiconductor metasurfaces

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Abstract: We develop neural network based equation learners to discover governing equations relating metasurfaceincoherent emission steering to critical features defining its spatial refractive index profiles.

The ability to achieve spatiotemporal control of incoherent light remains a critical impediment to several applications including LED based remote sensing and holographic displays. Traditional (phased array based) beam forming principles relying on spatiotemporal coherence of the light cannot be directly applied to incoherent light. Recently light emitting metasurfaces – made up of sub-wavelength array of optical resonators with embedded light emitters (quantum well [1] or quantum dots [2]) – have demonstrated the ability to statically and dynamically steer incoherent light emission. Metasurfaces realize control of incoherent light emission through spatial periodic structuring (momentum) of the size or refractive index of the resonators, resulting in low directivity (ratio of steered signal along the desired direction to the sum of steered signal across all directions) of the light emission. While metasurface steering of incoherent light emission based on momentum matching is in its infancy, the theoretical framework or numerical simulations required to model the far-field emission profile of a collection of incoherent emitters embedded in a metasurface remains expensive and impractical. Within this context, we leverage the potential of a reconfigurable light emitting metasurface as test-bed to realize governing equations that describe the steering properties of incoherent light emitters under nearly arbitrary spatial refractive index profiles. Machine learning frameworks such as symbolic regression and neural networks [5] have clearly shown the ability to automatically discover equations. In this work, we combine the expressive capacity of neural networks with a neuron-based pruning scheme to sift through thousands of equations and discover parsimonious laws relating features in aperiodic pump patterns to the observed incoherent emission steering.

We recently demonstrated that by projecting a spatially structured (using a spatial light modulator, SLM) optical pump (at 800nm, 40fs pulses with 2-3mJ/cm^2 at 1 KHz repetition rate shown in Fig 1A) on a GaAs metasurface (Fig 1B), the incoherent light emission form the InAs quantum dots within the GaAs resonators can be dynamically steered. We implemented a closed-loop experimental setup, driven by a machine learning, to realize high efficiency steering of the light emission [3]. The machine learning framework consists of an active learning agent coupled with a variational autoencoder that generates new images on the SLM. The closed-loop experiments discovered that aperiodic pump intensity patterns which can be described as $y = (ax^2 + bx)\%2\pi$ resulted in high efficiency (Fig 1 C, D, E) steering of the incoherent light emission. These pump patterns can be thought of as a combination of spatial phase terms describing a lens (a) and grating (b) terms. We observe that high

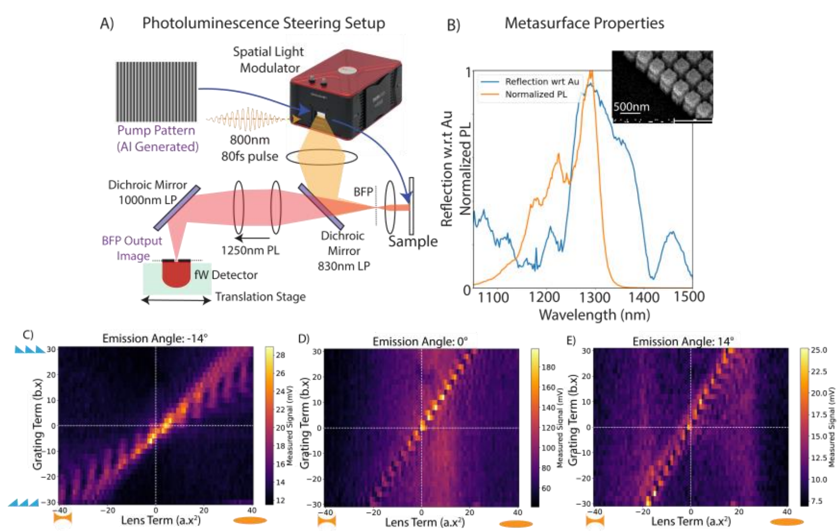

Figure 1: (A) Photoluminescence (PL) steering setup where an ultrafast 800nm optical pump is imaging an intensity pattern from a spatial light modulator on metasurface sample and collecting the far-field emission in the back focal plane (BFP) using a lock-in detector. (B) Metasurface properties of reflection (blue) and emission (orange) spectra with a scanning electron microscope image of the device shown in the inset. (C,D,E) Incoherent emission at various emission (-14, 0,+14) angles, as a function of characteristics of the pump pattern. We study pump patterns of the form: $y = ax^2 + bx$, varying a and b.

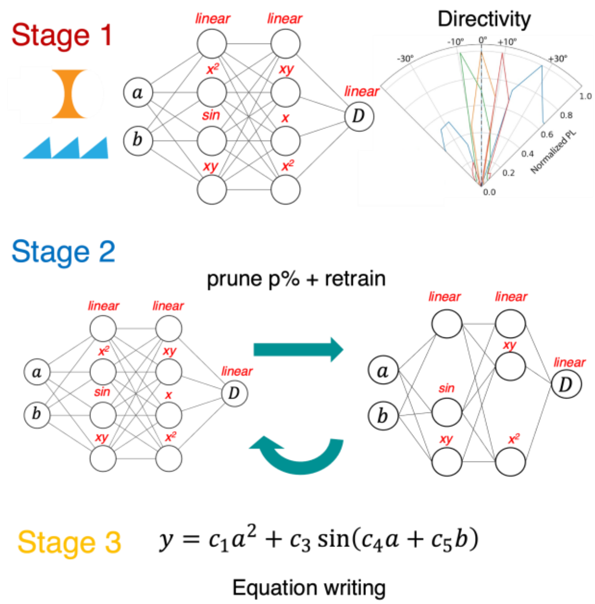

Figure 2: Equation learner framework: Our equation learner framework uses a customized neural network with physics-based activation functions defined for each neuron. Stage 1 first performs an initial fit to the dataset, establishing an acceptable error level. Stage 2 then iteratively prunes this network, removing neurons that have the lowest contribution in each layer. This pruning repeats until a desired level (e.g. 90%). In Stage 3, we write an (example) equation using the neural network's weights and activation functions.

intensity PL at specific steering angles requires specific combinations of a and b, with most other combinations resulting in limited steering. The specific steering intensity changes across various steering angles, but a consistent trend is observed where a linear subspace of the a, b space always shows high intensities which goes beyond our previous understanding of steering based solely on momentum matching principles.

We now wish to relate the emission directivity (D) to the critical features describing the spatial refractive profiles (a and b) via interpretable equations. We define a customized neural network (Figure 2) where each neuron has individual activation functions that are relevant to equations in the physical domain (sin, cos, product, addition) to obtain an equation describing the relationship between a, b and directivity, in three stages. In stage 1, we perform an initial fit of a network of pre-defined size (number of layers and neurons) to the dataset, which establishes a baseline accuracy for the network during the equation fitting process. In stage 2, we iteratively prune the network using an automated scheme that periodically removes a fixed percentage of least-contributing neurons in each layer. The network is re-trained before pruning occurs again, and this cycle continues until we reach a desired pruning level. This scheme gives us the most parsimonious equation that fits the dataset well [4], borrowing from Occam's razor. In our workflow, the equation learner network iterates over multiple choices of initial activation functions, pruning rates, and network sizes to arrive a set of pruned networks, from which, in Stage 3, we write an equation using the weights and activation functions in the network that describes the dataset the best (lowest error), while maintaining parsimony.

Below is one of the equations that we discovered relates steering intensity to the a and b parameters that describe our pump pattern. $D = 21.34b^2(b^2 - 0.7) - 147.44\eta^2 - 8.98(b - 0.58\sin(3.22b))^2 + 1059.22(\eta^2 - 0.01\sin(3.75b))^2 + 2.29\sin(2.79b) + 14.45$; where $\eta = 0.67b - a$. The dominant presence of η indicates that a specific combination of a and b affect steering the most, as Fig 1D would suggest. The appearance of $(0.67b - a)^4$ also indicates that both a and b contribute equally to incoherent emission steering, which extrapolates our understanding of incoherent emission steering beyond momentum matching defined solely on the grating term (b). While this equation is one of the many equations we discovered, this clearly demonstrates the power of our equation learner framework to formalize intuitions and data observations into rigorous equations. This is a generalizable approach and has significant implications in automated discovery of equations describing spatiotemporal control of incoherent emission, especially in combination with self-driving labs and autonomous experiments for rapid data collection.

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